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Exploring Links Between Physical and Probabilistic Models of Volcanic Eruptions: The Soufriere Hills Volcano, Montserrat

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Exploring links between physical and probabilistic models of volcanic eruptions: The Soufrière Hills Volcano, Montserrat

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[1] Probabilistic methods play an increasingly important role in volcanic hazards forecasts. Here we show that a probability distribution characterized by competing processes provides an excellent statistical fit (>99% confidence) to repose intervals between 75 vulcanian explosions of Soufrière Hills Volcano, Montserrat in September–October, 1997. The excellent fit is explained by a physical model in which there are competing processes operating in the upper volcano conduit on different time scales: pressurization due to rheological stiffening and gas exsolution, and depressurization due to development of permeability and gas escape. Our experience with the Soufrière Hills Volcano eruption sequence suggests that volcanic eruption forecasts are improved by accounting for these different conduit processes explicitly in a single probability model. *INDEX TERMS:* 8419 Volcanology: Eruption monitoring (7280); 8414 Volcanology: Eruption mechanisms; 8499 Volcanology: General or miscellaneous. **Citation:** Connor, C. B., R. S. J. Sparks, R. M. Mason, C. Bonadonna, and S. R. Young, Exploring links between physical and probabilistic models of volcanic eruptions: The Soufrière Hills Volcano, Montserrat, *Geophys. Res. Lett.*, *30*(13), 1701, doi:10.1029/2003GL017384, 2003.

1. Introduction

[2] *Wickman* [1966] suggested that analysis of volcano repose intervals can be used to forecast eruptions. Subsequently, several types of probability distributions have been used to model repose intervals. These include classical failure models, such as the Weibull distribution [*Ho*, 1996], that assume the probability of eruptions increases exponentially, or in an accelerated exponential fashion, as the time since that last eruption increases. This type of failure model is attractive because the timing and mechanics of volcanic eruptions have been described in terms of a materials failure model [*Voight*, 1988, 1989; *Cornelius and Voight*, 1994], which results in a Weibull distribution of repose intervals [*Weibull*, 1951]. Power-law fits [*Pyle*, 1998] have also been used to model repose intervals. Both Weibull and power law models,

however, commonly fail to explain significant variation in eruption repose interval data. This is a problem because extreme cases, such as the probability of comparatively long- or short-repose intervals, are often most relevant in volcanology [*Woo*, 1999; *Newhall and Hoblitt*, 2002; *Connor et al.*, 2001; *Sparks*, 2003]. For example, what repose interval is sufficiently long to conclude eruptive activity has ceased altogether? Is there a time interval immediately after eruptions when it is reasonably safe to work near the volcano? Volcanologists must be able to address these and similar concerns with confidence using probabilistic methods.

[3] We have found that models of competing processes, usually referred to as log logistic probability distributions, successfully explain all variation in repose interval data with a high degree of confidence. In log logistic models, some parameters work to increase the probability of an event with time, while others, commonly operating on different time scales, work to diminish the probability. Such competing parameters are common in nature. In volcanic conduits and domes, eruptions occur when pressure generated by gas exsolution exceeds the mechanical strength of magma and overlying rock. Factors that work to build pressure in these environments include gas exsolution and rheological stiffening of the magma. Other factors can work to relieve pressure, such as the development of fractures and interconnected bubble networks that increase permeability and aid gas escape. In the following, we argue that application of the log logistic model to Soufrière Hills Volcano repose interval data is successful precisely because the probability model is linked to these geologically meaningful parameters.

2. The 1997 Vulcanian Explosions

[4] The andesitic Soufrière Hills Volcano, situated in southern Montserrat in the Lesser Antilles island arc, started to erupt on 18 July 1995 [*Robertson et al.*, 1998]. Although the eruption has predominantly involved dome growth and generation of pyroclastic flows by dome collapse, there have also been episodes of explosive activity: a single explosion on 17 September 1996, 13 vulcanian explosions between 4 to 13 August 1997 and 75 explosions between 22 September to 21 October 1997 [*Robertson et al.*, 1998; *Young et al.*, 1999; *Druitt et al.*, 2002]. The last episode (September–October 1997) is considered here. During this episode, eruptions were characterized by strong, energetic
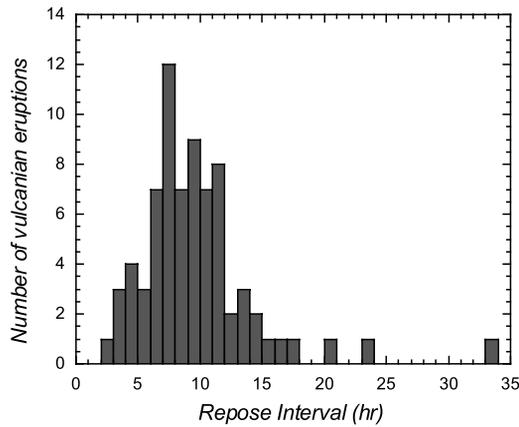
Figure 1. Histogram of 74 repose intervals between vulcanian explosions at Soufrière Hills volcano between September–October, 1997. Data from *Young et al.* [1999].

explosions: eruption columns ascended between 5 to 15 km a.s.l. with individual ejecta volumes up to 6.6×10^5 m^3. All but two of the explosions generated pyroclastic flows by fountain collapse [*Druitt et al.*, 2002]. Each of the vulcanian explosions started with an intense phase of peak discharge of a few tens of seconds, followed by a low-intensity phase of weak, pulsatory venting of gas and tephra lasting a few tens of minutes. Repose intervals between explosions varied from 2.77 to 33.7 hrs, with a median repose interval of 9.0 hr and mean 9.6 ± 0.5 hr (Figure 1).

3. Probability Analysis

[5] Here we analyze the repose interval data, the repose interval being defined as the time elapsed between the onset of each explosion picked from seismic records. The survivor function, $S_T(t)$, gives the probability of a repose interval T exceeding some time t:

$$S_T(t) = P[T \geq t] \tag{1}$$

From the set of observations, $S_T(t)$ is calculated by putting the set of repose intervals in rank order so that $T_{i=0} < T_1 < \ldots < T_{N1}$, where N is the total number of repose intervals ($N = 74$, Figure 1). Then:

$$S_{T,obs}(t_i) = \frac{N - i}{N} \tag{2}$$

Our aim is to estimate $S_T(t)$ from continuous probability distributions and identify distributions that give a good fit.

[6] Voight and colleagues [*Voight*, 1988, 1989; *Cornelius and Voight*, 1994] suggested that eruptions follow a material failure law that can be expressed as:

$$\frac{d\Omega}{dt} = At^k \tag{3}$$

where Ω is an observable quantity such as deformation, tilt, or seismic energy release that varies as a function of time, t. Alternatively, Ω could be defined as a physical property of the volcano, such as pressure in the upper part of the conduit. Here, the time of eruption is defined as the

approximate time when Ω reaches some critical value, Ω_c. Parameters A and k are estimated from observations of Ω with time. The parameter A [equation (3)] is a rate constant. The value of k has intrinsic physical significance for materials and can be estimated experimentally:

$$\frac{d^2\Omega}{dt^2} = C\left(\frac{d\Omega}{dt}\right)^\alpha \tag{4}$$

Thus, both α and C are estimated from, say, a graph of measured creep acceleration as a function of creep rate during the terminal stages of strain before failure. The parameters k and α are related by:

$$k = \frac{1}{\alpha - 1} \tag{5}$$

A wide variety of data, including seismic energy release, deformation, and tilt data [*Cornelius and Voight*, 1994] indicates that $1 < \alpha < 2$.

[7] *Weibull* [1951] used a similar expression to forecast failure rates of materials, suggesting that a survivor function should have the form:

$$S_T(t) = \exp\left[-(t/\hat{\mu})^k\right] \tag{6}$$

This is identical to the expression proposed by *Voight* [1988] if $A = 1/\hat{\mu}^k$, and the parameter $\hat{\mu}$ is estimated from the observed distribution. For Weibull distributions, $\hat{\mu}$ is the mean; for the log logistic distribution, $\hat{\mu}$ is the median [*Cox and Oakes*, 1984]. The Weibull probability density function [*Cox and Oakes*, 1984] is:

$$f_T(t) = \left(\frac{k}{\hat{\mu}}\right)\left(\frac{t}{\hat{\mu}}\right)^{k-1} \exp\left[-(t/\hat{\mu})^k\right] \tag{7}$$

The ratio of $f_T(t)$ and $S_T(t)$ is an important indicator of hazard over some comparably short time interval:

$$h(t) = \frac{f_T(t)}{S_T(t)} = \frac{k}{\hat{\mu}}\left(\frac{t}{\hat{\mu}}\right)^{k-1} \tag{8}$$

The hazard function, $h(t)$, can be thought of as an instantaneous recurrence rate, giving a "local" estimate of the probability of volcanic eruptions during some time interval since the last event:

$$P[N \geq 1, t < T < t + \Delta t | N = 0, T < t] = 1 - \exp[-\Delta t \cdot h(t)] \tag{9}$$

where Δt is the time interval. In the special case $\alpha = 2$ ($k = 1$), the probability density function is purely exponential, $h(t) = 1/\hat{\mu}$, and the probability of eruption does not change with time. For the Weibull distribution (with $1 < \alpha < 2$) and probability of eruptions increases with time for increasingly long periods of repose.

[8] Comparison of the observed survivor function and $S_T(t)$ for $\hat{\mu} = 9.6$ hr and varying values of k reveals some fundamental properties and shortcomings of the Weibull model (Figure 2a). Based on previous work [*Voight*, 1988] and high temperature rock creep experiments for Soufrière Hills Volcano dome rock [*Sparks et al.*, 2000], we considered the range $1 < k < 5$. For an accelerated failure model (i.e., $k > 1$) eruptions are less frequent than estimated by a
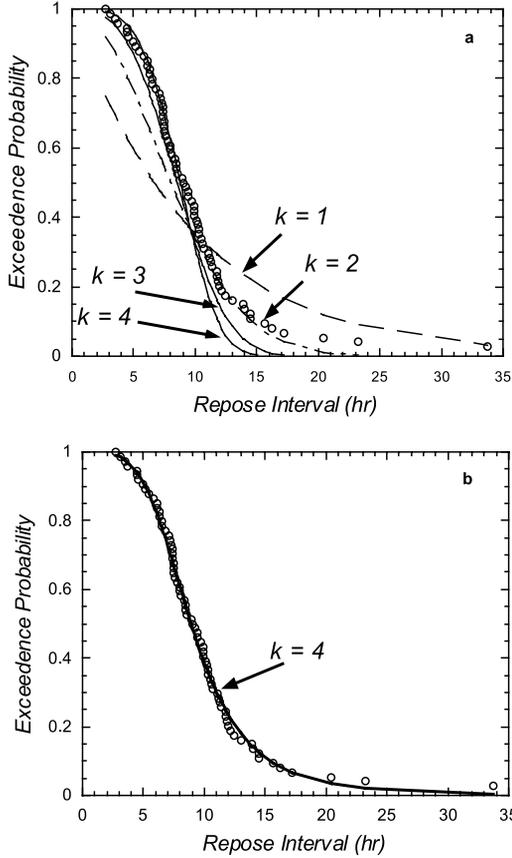
Figure 2. (a) Comparison of the observed survivor function (open circles) and solutions using the Weibull model, equation (6), for $\hat{\mu} = 9.6$ (observed distribution mean) and various values of k. Using $k = 4$, estimated from experimental data, gives a good fit to the observed distribution at $t < \hat{\mu}$. The exponential model corresponds to $k = 1$, and does not fit the observed distribution. (b) The observed survivor function is fit with >99% confidence using the log logistic function, (equation (13b), with $\hat{\mu} = 9.0$ hr (observed distribution median) and $k = 4$.

purely exponential model ($k = 1$) at times much less than $\hat{\mu}$. The Weibull distribution provides an excellent fit to the observed repose interval data for $T < \hat{\mu}$. When excellent fit to the observed repose interval data for $T > \hat{\mu}$, however, the Weibull model diverges from the observed distribution and more eruptions occurred at longer repose intervals than predicted (Figure 2a). Considering the entire distribution of repose intervals (Figure 2a), the observed distribution is not Weibull with >90 % confidence (two-tailed Kolmogorov-Smirnov test). Furthermore, errors in the fit are positively autocorrelated, especially in the tails of the distribution (Durbin-Watson statistic <0.03). Overall, the Weibull model is inappropriate for these repose interval data.

[9] The log logistic equation provides an alternative model of repose intervals that has the desirable property of limiting $\frac{d\Omega}{dt}$ for large values of t:

$$t\frac{d\Omega}{dt} = k\Omega - \frac{k}{\Omega_{eq}}\Omega^2 \tag{10}$$

where k and Ω are defined as above. The term Ω_{eq} represents an equilibrium condition that inhibits $\frac{d\Omega}{dt}$. Equation (10) can be solved for Ω/Ω_{eq}:

$$\frac{\Omega}{\Omega_{eq}} = \frac{1}{1 + (\Omega_{eq}/\Omega_0 - 1)(t/t_0)^{-k}} \tag{11}$$

where Ω_o is some initial pressure at time $t_o = 1$.

[10] Density functions can be written in their various forms: $f_T(t)$, $S_T(t)$, and $h(t)$. Taking:

$$\hat{\mu} = t_o\left[\frac{\Omega_{eq}}{\Omega_o} - 1\right]^{1/k} \tag{12}$$

Then [*Cox and Oakes*, 1984]:

$$f(t) = \frac{kt^{k-1}\left(\frac{1}{\hat{\mu}}\right)^k}{\left[1 + \left(\frac{t}{\hat{\mu}}\right)^k\right]^2} \tag{13a}$$

$$S(t) = \frac{1}{1 + \left(\frac{t}{\hat{\mu}}\right)^k} \tag{13b}$$

$$h(t) = \frac{kt^{k-1}\left(\frac{1}{\hat{\mu}}\right)^k}{1 + \left(\frac{t}{\hat{\mu}}\right)^k} \tag{13c}$$

The log logistic survivor function $S_T(t)$ fits the data for $k = 4$ ($\alpha = 1.25$) and $\hat{\mu} = 9.0$ hr, with greater than 99% confidence (Kolmogorov-Smirnov test)(Figure 2b).

4. Discussion

[11] We propose a physical basis to account for the success of the log logistic distribution. The vulcanian explosions are attributed to the build up of gas pore pressure in the upper conduit to a tensile strength threshold where the explosive fragmentation initiates [*Druitt et al.*, 2002; *Alidibirov and Dingwell*, 1996; *Melnik and Sparks*, 2002]. Pressure increase can occur both by rheological stiffening of degassing and crystallizing magma, resulting in greater resistance and dynamic pressure in the upper conduit [*Voight et al.*, 1999; *Sparks*, 1997; *Melnik and Sparks*, 1999], and by gas exsolution from supersaturated melt. However, gas can escape and pore pressure can be reduced if the magma becomes permeable by gas bubble coalescence or fracture network development [*Taylor et al.*, 1983; *Jaupart*, 1998; *Sparks et al.*, 2000]. This model is supported by observations from the August 1997 episode of vulcanian explosions when ground inflation and hybrid earthquakes indicated pressure build up in the conduit leading to an explosion [*Druitt et al.*, 2002; *Voight et al.*, 1999]. Deflation accompanied by ash venting indicates gas release and pressure loss [*Voight et al.*, 1999; *Watson et al.*, 2000]. These latter processes also took place at the peak and end of tilt cycles that were not associated with an explosion [*Voight*

et al., 1999; *Watson et al.*, 2000]. In these cases the pressure maximum did not reach the threshold required to trigger an explosion, due to the overburden pressure from the overlying dome. Thus, there is evidence of competing processes at work to increase and decrease pressure in the conduit.

[12]   If Ω is taken as gas pore pressure in the upper part of the conduit, Ω_{eq} represents the equilibrium pressure where the rate of pressure increase due to degassing and crystallization is exactly balanced by the rate of pressure release, due to development of permeability and gas escape. Both of these pressures vary with depth and time in the conduit. If Ω/Ω_{eq} reaches unity at some depth in the conduit, with Ω_{eq} being less than the critical tensile strength of the magma, pressurization due to rheological stiffening and gas exsolution is balanced by gas loss, and a vulcanian explosion will not occur. As Ω/Ω_{eq} reaches unity asymptotically [equation (11)], longer repose intervals are expected as Ω_c approaches Ω/Ω_{eq}, corresponding to decreasing values of $h(t)$ [equation (13c)].

[13]   Based on the log logistic model, the probability of an eruption after a repose interval of 40 hr is $P[T > 40\ hr] = 0.002$. While this probability is low, it does not appear to be sufficiently low to conclude that the eruption sequence is over [c.f., *Pyle*, 1998]. Given that 74 eruptions had already occurred, and uncertainty in k and $\hat{\mu}$, a probability of 0.0001, or $T > 85\ hr$ might be better suited to consider the eruption sequence over and the model no longer applies. The motivation for this conservatism is clearly illustrated by application of equation (9). If the eruption sequence is not over, and 40 hr have passed since the last eruption, the probability of an eruption in the next hour is high - $P[N = 1, 40 < T < 41\ hr \mid N = 0, T < 40\ hr] = 0.095$.

[14]   For Soufrière Hills Volcano, we are able to recognize the log logistic pattern of repose intervals because of the comparatively large number of vulcanian explosions. In the absence of such a robust data set, physical models and observations may provide estimates of distribution parameters. Recent models of the dynamics of magma flows in conduits suggest that competing processes, namely gas bubble pressurization and development of permeability in the vesiculating magma, are a common feature of erupting volcanoes [*Alidibirov and Dingwell*, 1996; *Melnik and Sparks*, 2002]. Provided that the key parameters of the probability density function, $\hat{\mu}$ and k, can be estimated from an improved understanding of conduit processes (e.g., equations (10)–(12)), or observations [c.f., *Voight*, 1989; *Voight et al.*, 1999], the log logistic model may have wide application as an eruption-forecasting tool.

References

Alidibirov, M. A., and D. B. Dingwell, High-temperature fragmentation of magma by rapid decompression, *Nature*, *380*, 146–148, 1996.

Connor, C. B., B. E. Hill, B. Winfrey, N. W. Franklin, and P. C. La Femina, Estimation of volcanic hazards from tephra fallout, *Natural Hazards Review*, *2*, 33–42, 2001.

Cornelius, R. R., and B. Voight, Seismological aspects of the 1989–1990 eruption of Redoubt volcano, Alaska: The materials failure forecast method (FFM) with RSAM and SSAM seismic data, *J. Volcanology and Geothermal Research*, *62*, 469–498, 1994.

Cox, D. R., and D. Oakes, *Analysis of Survival Data*, Chapman and Hall, London, 201 pp., 1984.

Druitt, T. H., S. R. Young, B. Baptie, C. Bonadonna, E. S. Calder, A. B. Clarke, P. D. Cole, C. L. Harford, R. A. Herd, R. Luckett, G. Ryan, and B. Voight, Episodes of cyclic Vulcanian explosive activity with fountain collapse at Soufrière Hills Volcano, Montserrat, in *The Eruption of Soufrière Hills Volcano, Montserrat, from 1995 to 1999*, edited by T. H. Druitt and B. P. Kokelaar, Geological Society, London, Memoir, *21*, 281–306, 2002.

Ho, C.-H., Volcanic time-trend analysis, *J. Volcanology and Geothermal Research*, *74*, 171–177, 1996.

Jaupart, C., Gas loss from magmas though conduit walls during eruption, in *The Physics of Explosive Volcanic Eruptions*, edited by J. S. Gilbert and R. S. J. Sparks, Special Publication of the Geological Society, London, *145*, 73–90, 1998.

Melnik, O., and R. S. J. Sparks, Non-linear dynamics of lava dome extrusion, *Nature*, *402*, 37–41, 1999.

Melnik, O., and R. S. J. Sparks, Modelling of conduit flow dynamics during explosive activity at Soufrière Hills, Volcano, Montserrat, in *The Eruption of Soufrière Hills Volcano, Montserrat, from 1995 to 1999*, edited by T. H. Druitt and B. P. Kokelaar, Geological Society, London, Memoir, 21, 307–317, 2002.

Newhall, C. G., and R. P. Hoblitt, Constructing event trees for volcanic crises, *Bul. Volcanol.*, *64*, 3–20, 2002.

Pyle, D. M., Forecasting sizes and repose times of future extreme volcanic events, *Geology*, *26*, 367–370, 1998.

Robertson, R. E., P. Cole, R. S. J. Sparks, C. Harford, A. M. LeJeune, W. J. McGuire, A. D. Miller, M. D. Murphy, G. E. Norton, N. F. Stevens, and S. R. Young, The explosive eruption of the Soufrière Hills Volcano, Montserrat, West Indies, 17 September, 1996, *Geophys. Res. Lett.*, *25*, 3429–3432, 1998.

Sparks, R. S. J., Causes and consequences of pressurization in lava dome eruptions, *Earth Planet. Sci. Lett.*, *350*, 177–189, 1997.

Sparks, R. S. J., Forecasting volcanic eruptions, *Earth Planet. Sci. Lett.*, in press, 2003.

Sparks, R. S. J., M. D. Murphy, A. M. LeJeune, R. B. Watts, J. Barclay, and S. R. Young, Control on the emplacement of the andesite dome of the Soufrière Hills Volcano, Montserrat, by degassing-induced crystallization, *Terra Nova*, *12*, 14–20, 2000.

Taylor, B. E., J. C. Eichelberger, and H. R. Westrich, Hydrogen isotopic evidence of rhyolitic magma degassing during shallow intrusion and eruption, *Nature*, *306*, 541–545, 1983.

Voight, B., A method for prediction of volcanic eruptions, *Nature*, *332*, 125–130, 1988.

Voight, B., A relation to describe rate-dependent material failure, *Science*, *243*, 200–203, 1989.

Voight, B., R. S. J. Sparks, A. D. Miller, R. C. Stewart, R. P. Hoblitt, A. Clarke, J. Ewart, W. P. Aspinall, B. Baptie, E. S. Calder, P. Cole, T. H. Druitt, C. Hartford, R. A. Herd, P. Jackson, A. M. LeJeune, A. B. Lockhart, S. C. Loughlin, R. Luckett, L. Lynch, G. E. Norton, R. Robertson, I. M. Watson, R. Watts, and S. R. Young, Magma flow instability and cyclic activity at Soufrière Hills Volcano, Montserrat, British West Indies, *Science*, *283*, 1138–1142, 1999.

Watson, I. M., C. Oppenheimer, B. Voight, P. W. Francis, A. Clarke, J. Stix, A. Miller, D. M. Pyle, M. R. Burton, S. R. Young, G. Norton, S. Loughlin, B. Darroux, and MVO Staff, The Relationship Between Degassing and Ground Deformation at Soufrière Hills Volcano, Montserrat, *J. Volcanol. Geotherm. Res.*, *98*, 117–126, 2000.

Wickman, F. E., Repose period patterns of volcanoes, *Ark. Mineral. Geol.*, *4*, 291–367, 1966.

Weibull, W., A statistical distribution of wide applicability, *J. Appl. Mech.*, *18*, 293–297, 1951.

Woo, G., *The Mathematics of Natural Catastrophes*, Imperial College Press, 292 pp., 1999.

Young, S. R., P. D. Cole, E. S. Calder, B. J. Baptie, C. Bonadonna, P. W. Francis, R. A. Herd, P. Jackson, S. C. Loughlin, R. Luckett, G. E. Norton, K. Rowley, R. S. J. Sparks, R. Watts, and MVO Staff, Dome collapse and vulcanian explosive activity, September to October, 1997, Montserrat Volcano Observatory, *MVO Special Report 5*, 1999.

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