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Transportation of Machinery through a Confined Space

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Transportation of Machinery through a Confined Space

Abstract

Our goal in this paper is to determine whether a packaging machine of negligible width will fit through a hallway and into a certain room (see page 4 for comments about negligible width). Given the width of the hallway and the room, we apply calculus to find the minimum length available between the two. We determine that the machine will successfully fit into the room.

Keywords

Assembly line, Package medicine, Negligible width

Figure 1

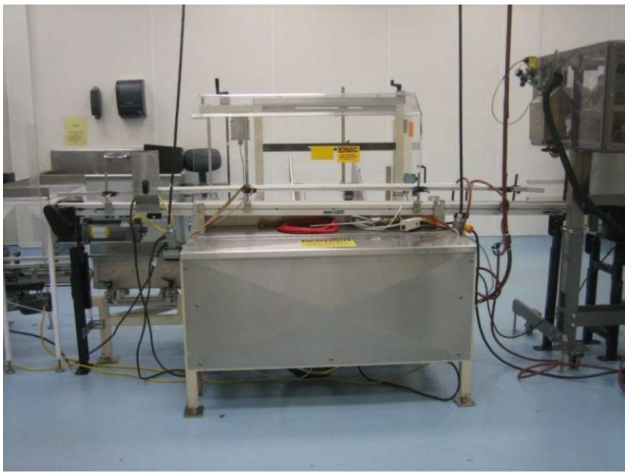
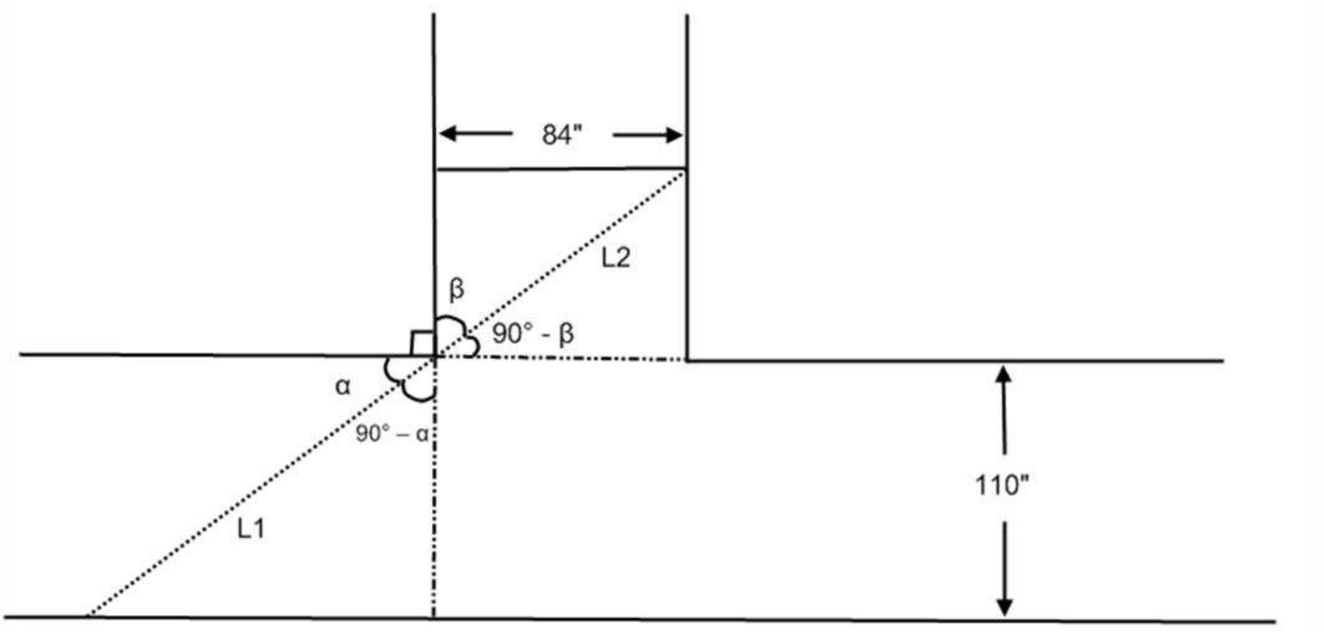


Figure 2

PROBLEM STATEMENT

A 20' long packaging machine of negligible width is ordered by a company and it must be determined whether the machine will fit around the corner of a hallway that is 110" wide into a room that is 84" wide. The negligibility of the width of the machine will be discussed on page 4.

Figure 3



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MOTIVATION

This problem was suggested by a Chemical Engineer working at a medicine packing facility and plant and was a problem he faced at work. The company ordered a new machine that is used to package medicine. This machine is 20 feet long and of negligible width. The engineer needs to determine whether the machine will fit through the plant hallways and into the designated room. Moving machinery is a key part of an engineer's job, especially in a packaging facility. Calculus and trigonometry are needed to solve this problem.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Consider the diagram representing the situation (Figure 3). The length of the area available around the hallway corner is represented by

$$L \coloneqq L_1 + L_2 = f(\alpha, \beta).$$

where L_1 and L_2 are measured in inches. Next, we identify the widths of the separate rooms and include them in the calculations:

$$L_1 \sin\alpha = 110$$

and

$$L_2 \sin\beta = 84.$$

The corner that the machine must maneuver around forms a 90 degree angle and therefore,

$$\alpha + \beta = 90°.$$

Rewriting these equations gives

$$L = \frac{110}{\sin\alpha} + \frac{84}{\sin\beta}.$$

Since $\beta = 90° - \alpha$,

$$L = \frac{110}{\sin\alpha} + \frac{84}{\cos\alpha}.$$

The problem requires that we find the minimum length available around the corner and then determine if the machine will fit this length. The length of the machine must be at most the minimum length available around the hallway. To find the minimum length, we take the derivative of L with respect to α and set it equal to zero. This gives

$$\frac{dL}{d\alpha} = 84\left(\frac{\sin\alpha}{\cos^2\alpha}\right) - 110\left(\frac{\cos\alpha}{\sin^2\alpha}\right) = 0.$$

https://digitalcommons.usf.edu/ujmm/vol1/iss2/1
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DISCUSSION

We solve this equation for α:

$$84\left(\frac{\sin\alpha}{\cos^2\alpha}\right) = 110\left(\frac{\cos\alpha}{\sin^2\alpha}\right) \Rightarrow \tan^3\alpha = \frac{55}{42}, \quad \alpha = \arctan\left(\sqrt[3]{55/42}\right) \cong 47.6°.$$

Hence

$$\beta \cong 90° - 47.6° = 42.4°.$$

Since the machine is 20' long, its length in inches is 20ft · 12in/1ft = 240in. The minimum length around the corner must be greater than or equal to 240", or the machine will not fit through the door. The minimum length is

$$L = \frac{110}{\sin\alpha} + \frac{84}{\sin\beta} = \frac{110}{\sin 47.6°} + \frac{84}{\sin 42.4°} = 148.96 + 124.57 = 273.53.$$

The machine, therefore, will successfully fit through the hallway.

The initial problem was to determine whether a 20' machine of negligible width would fit around the corner of a 110" hallway and an 84" room. Designing an equation to express the angles of the hallway and the room corners and then taking the derivative of this equation allowed us to find the minimum length available for the machine to maneuver around the corner. The length of this area turned out to be 33.53" greater than the machine's length.

CONCLUSIONS AND RECOMMENDATIONS

We conclude that the machine will have plenty of clearance to fit into the room. This specific machine is designed to fit a 20' section of an assembly line of machines, and therefore the length cannot be compromised. The machine having a "negligible width" is a key factor in this problem. The bulk of the machine sits in the center (Figure 1), and the conveyor belt sticks out far beyond this center piece. Therefore, the 20' conveyor belt is the determining factor for the length. This conveyor belt is also only 4" wide, so the width of the machine—essentially the width of the conveyor belt—can be safely neglected.

If the machine had not successfully fit through the hallway, other options would need to be explored, such as having the machine assembled in place, or knocking down a wall to make room. Both of these options pose high cost for the company and are clearly not the most cost-efficient choices. Because of the engineer's ability to apply calculus, the safest and smartest option can be taken.

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